# Context-Free Grammars (CFGs) (1/3)

CFGs offer concise language specifications.

$$S ::= D \mid DS$$
$$D ::= 0 \mid 1$$

A grammar consists of:

- a set of non-terminals and a set of terminals
- a set of productions (here four):
  - "an $S$ may be replaced by $D$"
  - "an $S$ may be replaced by $DS$"
  - "a $D$ may be replaced by $0$"
  - "a $D$ may be replaced by $1$"
- A (non-terminal) start symbol $S$, typically written first.

# Context-Free Grammars (CFGs) (2/3)

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$$S ::= D \mid DS$$
$$D ::= 0 \mid 1$$

They do so by expressing a set of derivable strings.

Recipe: Repeatedly replace a non-terminal by a right-hand-side until there's only terminals left:

$$S \to D \to 0$$
$$S \to DS \to 0S \to 0D \to 01$$
$$S \to DS \to 1S \to 1DS \to 10S \to 10D \to 101$$

Q: Which language does this grammar represent?

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Q: Which language does this grammar represent?

Exercise: alter the grammar to yield base-10 digits

# Context-Free Grammars (CFGs) (3/3)

BNF grammars (short for 'Backus-Naur Form' after John Backus and Peter Naur) are a particular kind of CFGs.

A CFG can express a 'Context-Free Language'. These are strictly more powerful than, e.g., regular expressions.

Most language specifications today come with a CFG.

Q: Which languages do the following grammars define?

$S ::= 0 \mid 01 \mid 01S$ (terminals: $0$, $1$)

$S ::= ab \mid aSb$ (terminals: $a$, $b$)

$S ::= e \mid S + S \mid S - S$ (terminals: $e$, $+$, $-$)

$S ::= s \mid S\texttt{;}S \mid \texttt{if (}e\texttt{) \{ }S\texttt{ \}} \mid \texttt{while (}e\texttt{) \{ }S\texttt{ \}}$
(terminals: $s$, $e$, `;`, `if`, `while`, `(`, `)`, `{`, `}`)

# Other CFG examples

Grammars are used in real-world specifications:

A JSON grammar:

`https://www.json.org/json-en.html`

A grammar from the HTTP 1.1 RFC:

`https://tools.ietf.org/html/rfc2616#page-14`

The Java grammar from the Java Language Specification:

`https://docs.oracle.com/javase/specs/jls/se13/html/j`