Model-Based Testing, Part 1: QuickChecking Patricia Trees

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SM2-TES, Day 4

SDU

A bug's tale...

QuickCheck, briefly (Claessen-Hughes:00)

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Tests are described by a generator and a property.

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```
# QCheck_runner.run_tests ~verbose:true [mc91_const];;
generated error fail pass / total     time test name
[✗]   10    0    1    9 / 1000     0.0s McCarthy 91 constant

--- Failure --------------------------------------------------

Test McCarthy 91 constant failed (41 shrink steps):
```

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We can then test it according to the real specification:

```
let mc91_spec =
  Test.make ~name:"McCarthy␣91␣corr.␣spec" ~count:1000
    small_signed_int (fun n -> if n <= 101
                               then mc n = 91
                               else mc n = n - 10)
```

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```
# QCheck_runner.run_tests ~verbose:true [mc91_spec];;
generated error fail pass / total     time test name
[✓] 1000    0    0 1000 / 1000     0.0s McCarthy 91 corr. spec
============================================================================
success (ran 1 tests)
```

Patricia trees (Morrison:68,Okasaki-Gill:98)

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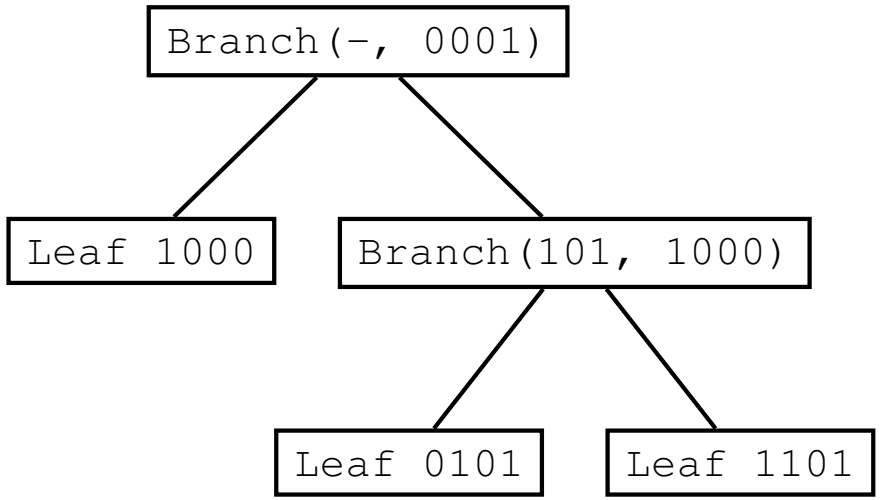

Traverse bits from LSB to MSB

`Branch(pre,brbit)` nodes:

- `pre` is a shared prefix
- `brbit` is a branching bit

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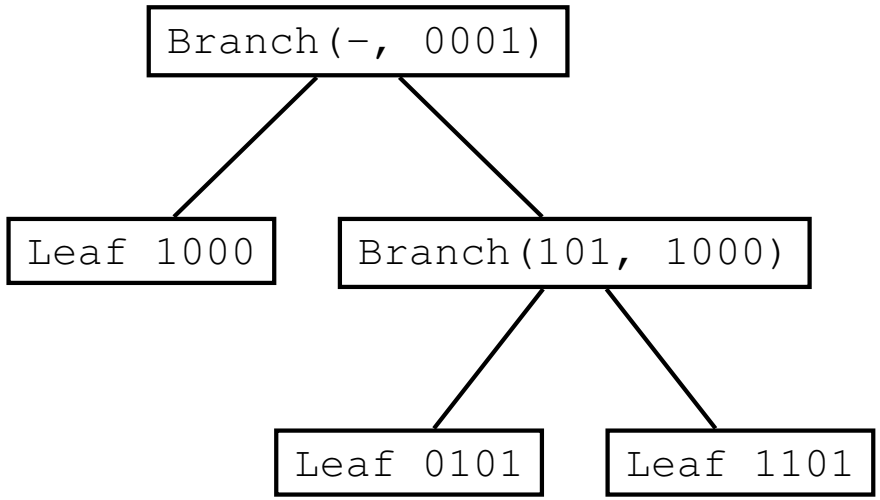

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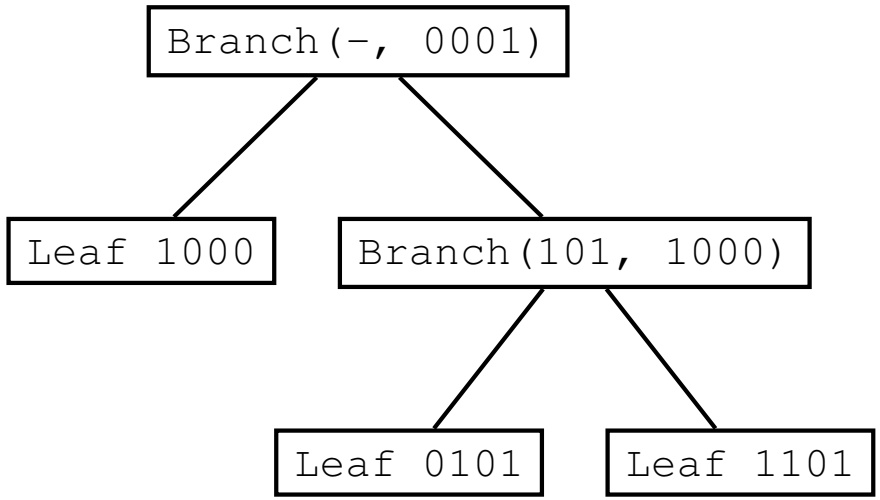

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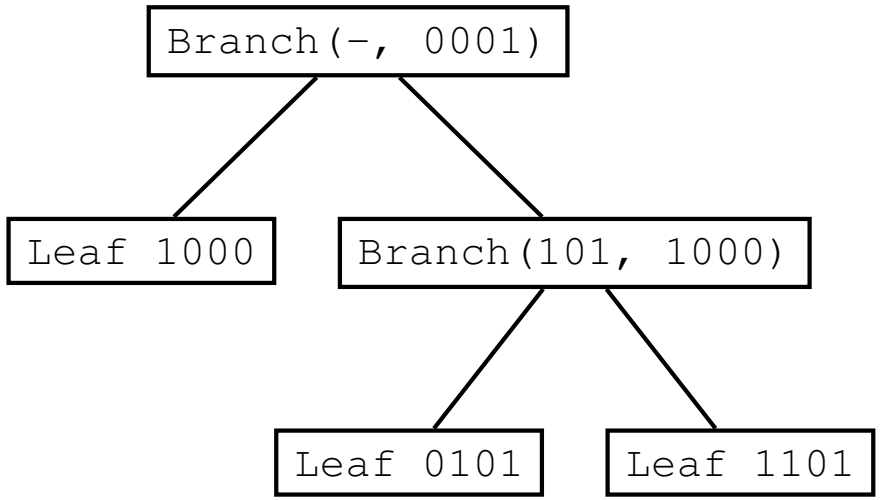

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`ptrees/ptset` is a popular OCaml implementation, which is used by JavaLib/SawJa, which is again used by Facebook's Infer analyzer.

What could possibly go wrong?

Non-trivial case analysis and bit fiddling in `ptset`:

```
let rec merge = function
  | t1,t2 when t1==t2 -> t1
  | Empty, t  -> t
  | t, Empty  -> t
  | Leaf k, t -> add k t
  | t, Leaf k -> add k t
  | (Branch (p,m,s0,s1) as s), (Branch (q,n,t0,t1) as t) ->
      if m == n && match_prefix q p m then
        (* The trees have the same prefix. Merge the subtrees. *)
        Branch (p, m, merge (s0,t0), merge (s1,t1))
      else if m < n && match_prefix q p m then
        (* [q] contains [p]. Merge [t] with a subtree of [s]. *)
        if zero_bit q m then
          Branch (p, m, merge (s0,t), s1)
        else
          Branch (p, m, s0, merge (s1,t))
      else if m > n && match_prefix p q n then
        (* [p] contains [q]. Merge [s] with a subtree of [t]. *)
        if zero_bit p n then
          Branch (q, n, merge (s,t0), t1)
        else
          Branch (q, n, t0, merge (s,t1))
      else
        (* The prefixes disagree. *)
        join (p, s, q, t)
```

Spilling the Patricia tree beans . . .

There was a bug:

- in Filliâtre's `ptset` module code and
- in the Okasaki-Gill:ML98 paper

which went unnoticed for ∼19 years.

I found it using QuickCheck, aka. property-based testing.

In this rest of this talk I'll (attempt to) explain you how. . .

For more details see:

J. Midtgaard
QuickChecking Patricia Trees
TFP'17

Outline

- QuickCheck, briefly
- Patricia trees
- Spilling the beans
- The basic idea
- Building a model
  (datatype, interpreter, generator, shrinker, . . .)
- Revisiting generation
- The bug and a fix
- Model-based quickchecking
- Conclusion

The `ptset` API

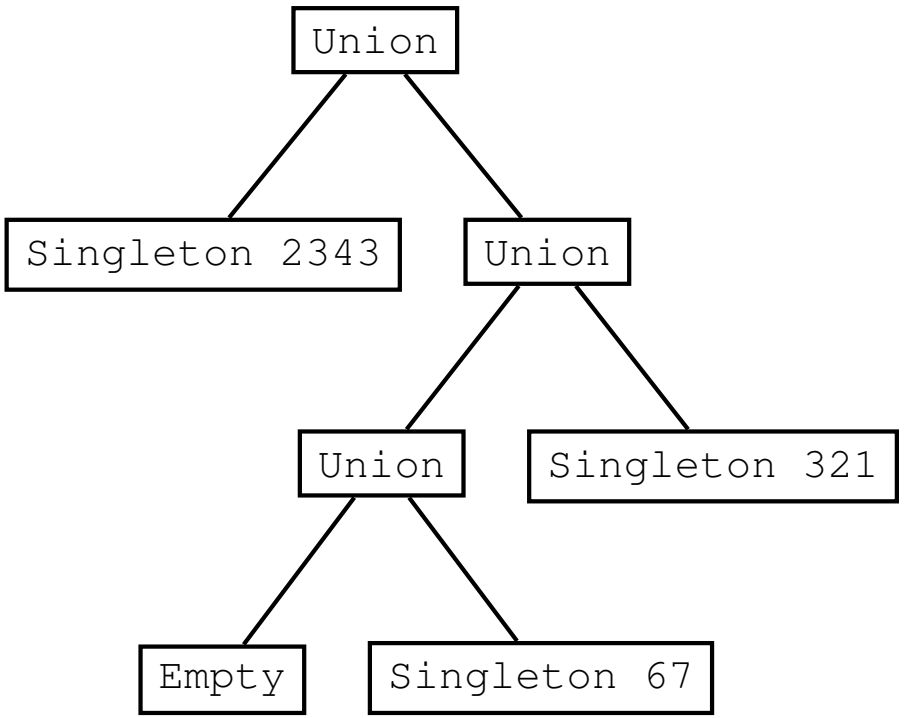
We consider the following subset of API operations:

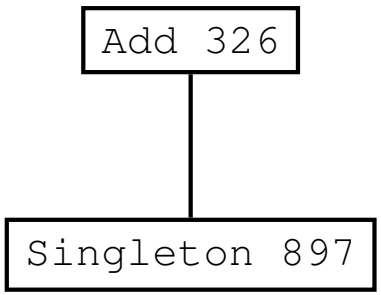
```
val empty     : Ptset.t
val singleton : int -> Ptset.t
val mem       : int -> Ptset.t -> bool
val add       : int -> Ptset.t -> Ptset.t
val remove    : int -> Ptset.t -> Ptset.t
val union     : Ptset.t -> Ptset.t -> Ptset.t
val inter     : Ptset.t -> Ptset.t -> Ptset.t
```

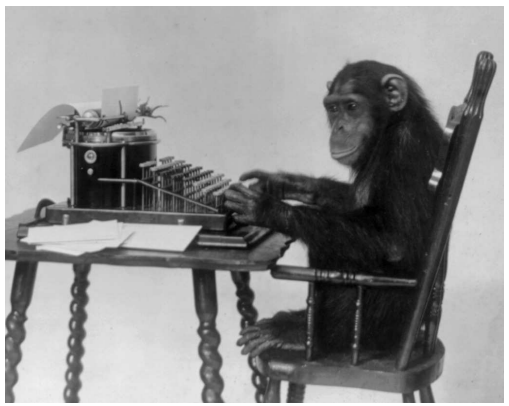
Each of them should be self-explanatory as set operations

The basic idea . . . (1/2)

The basic idea is to generate arbitrary trees:



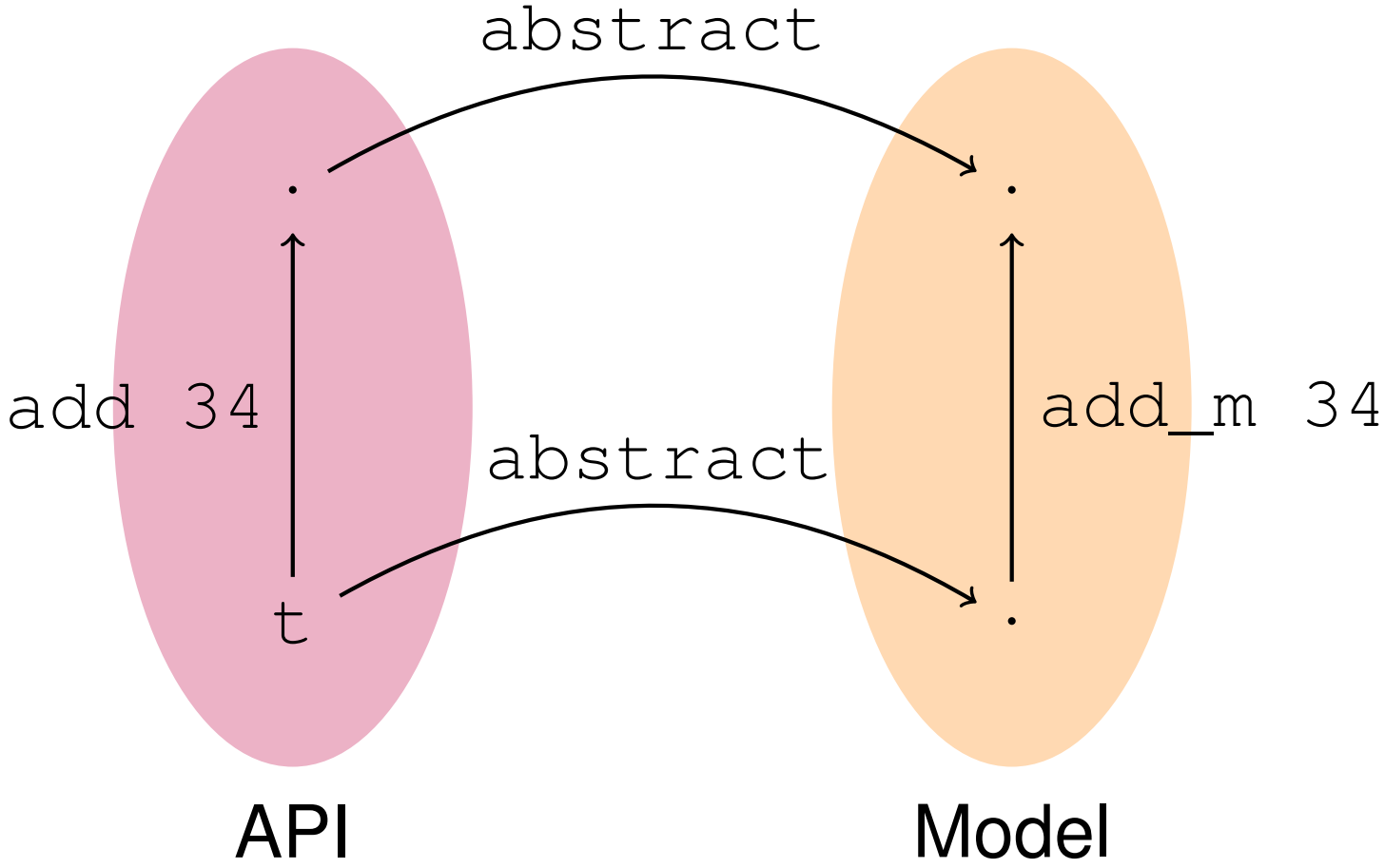




(apologies if they do not look sufficiently random)

The basic idea . . . (2/2)

and ensure a commuting diagram for each such tree `t`:



for a suitable model and `abstract` operation, and suitably generalized beyond the `add` operation.

A datatype for set operations

We declare a datatype to represent set operations symbolically:

```
type instr_tree =
  | Empty
  | Singleton of int
  | Add of int * instr_tree
  | Remove of int * instr_tree
  | Union of instr_tree * instr_tree
  | Inter of instr_tree * instr_tree
```

We exclude a symbolic `Mem` constructor as a query will not give rise to a new set.

API interpreter

It is straightforward to interpret trees over the API:

```
(*  interpret : instr_tree -> Ptset.t  *)
let rec interpret t = match t with
  | Empty          -> Ptset.empty
  | Singleton n    -> Ptset.singleton n
  | Add (n,t)      -> Ptset.add n (interpret t)
  | Remove (n,t)   -> Ptset.remove n (interpret t)
  | Union (t,t') ->
    let s  = interpret t in
    let s' = interpret t' in
    Ptset.union s s'
  | Inter (t,t') ->
    let s  = interpret t in
    let s' = interpret t' in
    Ptset.inter s s'
```

by mapping each symbolic node to the corresponding API operation (and suitable recursing).

A tree printer

It is straightforward to write a printer for the trees as a recursive decent:

```
(*  to_string : instr_tree -> string  *)
let rec to_string a = match a with
  | Empty          -> "Empty"
  | Singleton n    -> "Singleton " ^ (string_of_int n)
  | Add (n,t)      -> "Add (" ^ (string_of_int n) ^ ", "
                              ^ (to_string t) ^ ")"
  | Remove (n,t) -> "Remove (" ^ (string_of_int n) ^ ", "
                               ^ (to_string t) ^ ")"
  | Union (t,t') -> "Union (" ^ (to_string t) ^ ", "
                             ^ (to_string t') ^ ")"
  | Inter (t,t') -> "Inter (" ^ (to_string t) ^ ", "
                             ^ (to_string t') ^ ")"
```

This boilerplate could also have been auto-generated from the datatype definition.

A generator of arbitrary sets

```
(*  tree_gen : int Gen.t -> instr_tree Gen.t  *)
let tree_gen int_gen =
  Gen.sized (Gen.fix (fun rgen n -> match n with
    | 0 -> Gen.oneof [Gen.return Empty;
                      Gen.map (fun i -> Singleton i) int_gen]
    | _ ->
      Gen.frequency
        [(1,Gen.return Empty);
         (1,Gen.map  (fun i -> Singleton i) int_gen);
         (2,Gen.map2 (fun i t -> Add (i,t)) int_gen (rgen (n-1)));
         (2,Gen.map2 (fun i t -> Remove (i,t)) int_gen (rgen (n-1)));
         (2,Gen.map2 (fun l r -> Union (l,r)) (rgen (n/2)) (rgen (n/2)));
         (2,Gen.map2 (fun l r -> Inter (l,r)) (rgen (n/2)) (rgen (n/2)));
        ]))
```

This recursive generator is
- fueled (with an integer `n`) to guarantee termination
- weighted, to increase chance of certain nodes
- parameterized over the integer generator `int_gen`

A tree shrinker (based on iterators)

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(*  tshrink : instr_tree -> instr_tree Iter.t  *)
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  | Add (i,t) ->
    (Iter.of_list [Empty; t; Singleton i])
    <+> (Iter.map (fun t' -> Add (i,t')) (tshrink t))
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  | Remove (i,t) ->
    (Iter.of_list [Empty; t])
    <+> (Iter.map (fun t' -> Remove (i,t')) (tshrink t))
    <+> (Iter.map (fun i' -> Remove (i',t)) (Shrink.int i))
  | Union (t0,t1) ->
    (Iter.of_list [Empty;t0;t1])
    <+> (Iter.map (fun t0' -> Union (t0',t1)) (tshrink t0))
    <+> (Iter.map (fun t1' -> Union (t0,t1')) (tshrink t1))

  (* Inter case omitted *)
```

where <+> is an alias for `Iter.append`.

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let mem_m i s = List.mem i s
let add_m i s =
  if List.mem i s then s else List.sort compare (i::s)
let rec union_m s s' = match s,s' with
  | [], _ -> s'
  | _, [] -> s
  | i::is,j::js -> if i<j then i::(union_m is s') else
                     if i>j then j::(union_m s js) else
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                   if i>j then j::(union_m s js) else
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(* ... *)
```

Finally: `let abstract t = Ptset.elements t`

The agreement properties

We can now test agreement of each of the API entries:

```
let singleton_test =
  Test.make ~name:"singleton test" ~count:10000
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    (pair arb_tree arb_int)
    (fun (t,n) ->
      let s = interpret t in
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let union_test =
  Test.make ~name:"union test" ~count:10000
    (pair arb_tree arb_tree)
    (fun (t,t') ->
      let s  = interpret t in
      let s' = interpret t' in
      abstract (Ptset.union s s')
                          = union_m (abstract s) (abstract s'))
```

Finally: time for some testing!

Let's run our model-based tests:

```
random seed: 362256415
generated error  fail  pass / total     time test name
[✓]     1     0     0     1 /     1     0.0s empty
[✓] 10000     0     0 10000 / 10000     0.0s singleton test
[✓] 10000     0     0 10000 / 10000     0.1s mem test
[✓] 10000     0     0 10000 / 10000     0.1s add test
[✓] 10000     0     0 10000 / 10000     0.1s remove test
[✓] 10000     0     0 10000 / 10000     0.1s union test
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success (ran 7 tests)
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This is with `int_gen` generating integers uniformly.

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Seems to work!

If we repeat these 60.001 tests 10 times the implementation still seems to function correctly.

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Here's an attempt at a weighted integer generator:

```
let arb_int =
  frequency [(5,small_signed_int);
             (3,int);
             (1, oneofl [min_int;max_int])]
```

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  frequency [(5,small_signed_int);
             (3,int);
             (1, oneofl [min_int;max_int])]
```

This definition also has a reasonable chance of generating duplicate numbers.

Rerunning our tests. . .

Ooops:

```
--- Failure ------------------------------------------

Test union test failed (68 shrink steps):

(Add (-4611686018427387904, Singleton 0),
 Add (-4611686018427387904, Singleton 1))
```

We recognize `-4611686018427387904` as `min_int`.

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Over `Ptset` we actually get:

$$\{\texttt{min\_int}, 0\} \cup \{\texttt{min\_int}, 1\} = \{\texttt{min\_int}, 0, \texttt{min\_int}, 1\}$$

The problem (`union` just calls `merge`)

```
let rec merge = function
  | t1,t2 when t1==t2 -> t1
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  | Leaf k, t -> add k t
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      if m == n && match_prefix q p m then
        (* The trees have the same prefix. Merge the subtrees. *)
        Branch (p, m, merge (s0,t0), merge (s1,t1))
      else if m < n && match_prefix q p m then
        (* [q] contains [p]. Merge [t] with a subtree of [s]. *)
        if zero_bit q m then
          Branch (p, m, merge (s0,t), s1)
        else
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          Branch (p, m, s0, merge (s1,t))
      else if m > n && match_prefix p q n then
        (* [p] contains [q]. Merge [s] with a subtree of [t]. *)
        if zero_bit p n then
          Branch (q, n, merge (s,t0), t1)
        else
          Branch (q, n, t0, merge (s,t1))
      else
        (* The prefixes disagree. *)
        join (p, s, q, t)
```

Don’t use signed comparisons over branching bits!

An elegant fix

When I showed the example to Filliâtre he provided an elegant fix within hours.

For the purposes of comparing branching bits, we can instead use this function:

```
let unsigned_lt n m = n >= 0 && (m < 0 || n < m)
```

Rerunning our tests on the fixed version finds no issues:

```
random seed: 175347559
generated error  fail  pass / total     time test name
[✓]     1     0     0     1 /     1     0.0s empty
[✓] 10000     0     0 10000 / 10000     0.0s singleton test
[✓] 10000     0     0 10000 / 10000     0.1s mem test
[✓] 10000     0     0 10000 / 10000     0.1s add test
[✓] 10000     0     0 10000 / 10000     0.1s remove test
[✓] 10000     0     0 10000 / 10000     0.1s union test
[✓] 10000     0     0 10000 / 10000     0.1s inter test
```

Model-based quickchecking in general

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Few other/typed state-machine frameworks exist.

I've only recently learned that ScalaCheck has one.

Conclusion

I've presented

- a pedagogical application of QuickCheck
- with a surprising outcome: a 19 year old bug in Patricia trees
- a success story using the familiar FP tools.

The example underlines the importance of generators with QuickCheck.

Simon Cruanes has since reused the model to test another Patricia tree implementation (from `CC`).

The testing code is available online:

`https://github.com/jmid/qc-ptrees`