SM2-TES: Functional Programming and Property-Based Testing, Day 5

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Last week

- A brief look at OCaml's module system
- Shrinking counterexamples
- Model-based testing (of Patricia trees)

Last lecture’s exercises

QuickCheck, recap

QuickCheck-wise we've covered:

- ☐ properties
- ☐ generators (type-directed)
- ☐ statistics (observing generator distributions)
- ☐ shrinking
- ☐ model-based testing (of functional data structures)
- ☐ ...

and worked with these concepts in the `QCheck` framework

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QuickChecking Stateful Code

From functional to stateful code

For a start we've tested primarily functional code, i.e., pure code without side-effects

This setup makes life easier: functions are typically small and their functionality is determined by their parameters

But code can also be stateful: its functionality is a mixture of parameters and some internal state

QuickCheck can also be used to test such code

To do so, we should be able to generate (or bring the code to) arbitrary states and formulate properties about the state (which is today's topic)

Example: a queue

Suppose we have an imperative queue with the following interface:

```
module MyQueue :
 sig
   type 'a t                            (* the type of queues *)
   val empty : unit -> 'a t             (* queue creation *)
   val pop   : 'a t -> unit             (* may throw exception *)
   val top   : 'a t -> 'a option        (* peek at the front *)
   val push  : 'a -> 'a t -> unit       (* add element *)
 end
```

This can be easily implemented, e.g.,

- from scratch or
- as an interface to the builtin `Queue` module

Using our imperative queue

For example, we can interact with our queue at the toplevel as follows:

```
# let q = empty ();;
val q : '_a t = <abstr>
# push 1 q;;
- : unit = ()
# push 2 q;;
- : unit = ()
# push 3 q;;
- : unit = ()
# top q;;
- : int option = Some 1
# pop q;;
- : unit = ()
# top q;;
- : int option = Some 2
```

Algebraic specification

Our queue, algebraically

We would expect a number of algebraic equivalences:

```
let q = empty () in
 let x = top q
```
=
```
let q = empty () in
 let x = None
```

```
let q = empty () in
 let () = push m q in
  let x = top q
```
=
```
let q = empty () in
 let () = push m q in
  let x = Some m
```

```
let () = push m q in
 let () = push n q in
  let x = top q
```
=
```
let () = push m q in
 let x = top q in
  let () = push n q
```

```
let q = empty () in
 let () = push m q in
  let () = pop q
```
=
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let q = empty ()
```

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let () = push m q in
 let () = push n q in
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```
=
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let () = push m q in
 let () = pop q in
  let () = push n q
```

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Testing imperative instructions

How can we test these properties of the queue?

Since it is an abstract data type (ADT), we can only observe it (and change it) through the provided interface

But how do we QuickCheck the interface
(the “getters” and “setters”)?

Formal semantics provides operational equivalence: two instruction sequences are indistinguishable no matter what context they are put in

So we just write a generator of all possible program contexts?

...almost!

Since a queue is an ADT, we can only observe it (and change it) through the official hooks

So it should be sufficient to write

- a generator of ADT contexts and
- the property of “equality of ADT observations”

We’ll generate well-formed queue instruction sequences

- that begin with `empty`,
- contains `push`, `pop`, and `top` instructions,
- any prefix contains at least as many `push` as `pop` instructions

Symbolic instructions

We can implement a symbolic type of commands:

```
type command =
  | Push of int         (* ~ let () = push n q   *)
  | Pop                 (* ~ let () = pop q      *)
  | Top                 (* ~ let x = top q       *)
  | Let of int option   (* ~ let x = None/Some n *)
```

and write an interpreter that map them to their meaning:

```
(*  interp : int MyQueue.t -> command list -> int option list *)
let rec interp q cmds = match cmds with
  | []              -> []
  | (Push n)::cmds' -> let () = MyQueue.push n q in
                       interp q cmds'
  | Pop::cmds'      -> let () = MyQueue.pop q in
                       interp q cmds'
  | Top::cmds'      -> let e = MyQueue.top q in
                       e::(interp q cmds')
  | (Let x)::cmds'  -> x::(interp q cmds')
```

namely, a “list of observations”

From symbolic instructions to strings

QCheck (again) needs `to_string` coercion to print counterexamples:

```
(*  to_string : command -> string  *)
let to_string a = match a with
  | Push n -> "(Push␣" ^ (string_of_int n) ^ ")"
  | Pop    -> "Pop"
  | Top    -> "Top"
  | Let x  -> (match x with
               | None -> "(Let␣None)"
               | Some i -> "(Let␣(Some␣" ^ (string_of_int i) ^ "))")
```

Based on `to_string` we can write a version for lists of commands:

```
(*  commands_to_string : command list -> string *)
let commands_to_string = Print.list to_string
```

Number of queue elements

To only generate well-formed sequences, we need to keep track of the number of elements in a queue.

For this purpose, the following function is handy:

```
(*  delta : command list -> int  *)
let rec delta cmds = match cmds with
  | [] -> 0
  | (Push _)::cmds' -> (delta cmds') + 1
  | Pop::cmds'      -> (delta cmds') - 1
  | Top::cmds'      -> (delta cmds')
  | (Let _)::cmds'  -> (delta cmds')
```

It computes the element count change of a sequence of instructions

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Generator, take 1

Here's a first attempt at a pure generator:

```
(*  commands : int -> command list Gen.t  *)
let rec commands num =
  Gen.oneof
    ([ Gen.return [];
       Gen.map2 (fun i cmds -> (Push i)::cmds)
                                  Gen.int (commands (num+1));
       Gen.map (fun cmds -> Top::cmds) (commands num) ]
     @ if num = 0
       then []
       else [ Gen.map (fun cmds -> Pop::cmds)
                                     (commands (num-1)) ])
```

It tracks the element count using a parameter `num`

and chooses between 3 or 4 different generators

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Generator, take 1 – diverges!

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Generator, take 2 – guarded by parameter

Utilize `'a Gen.t = Random.State.t -> 'a` :

```
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```

Now the recursive calls are delayed to generation time

– yet we can still use module `Gen`'s combinators

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Generator, take 2 – guarded by parameter

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```

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– yet we can still use module `Gen`'s combinators

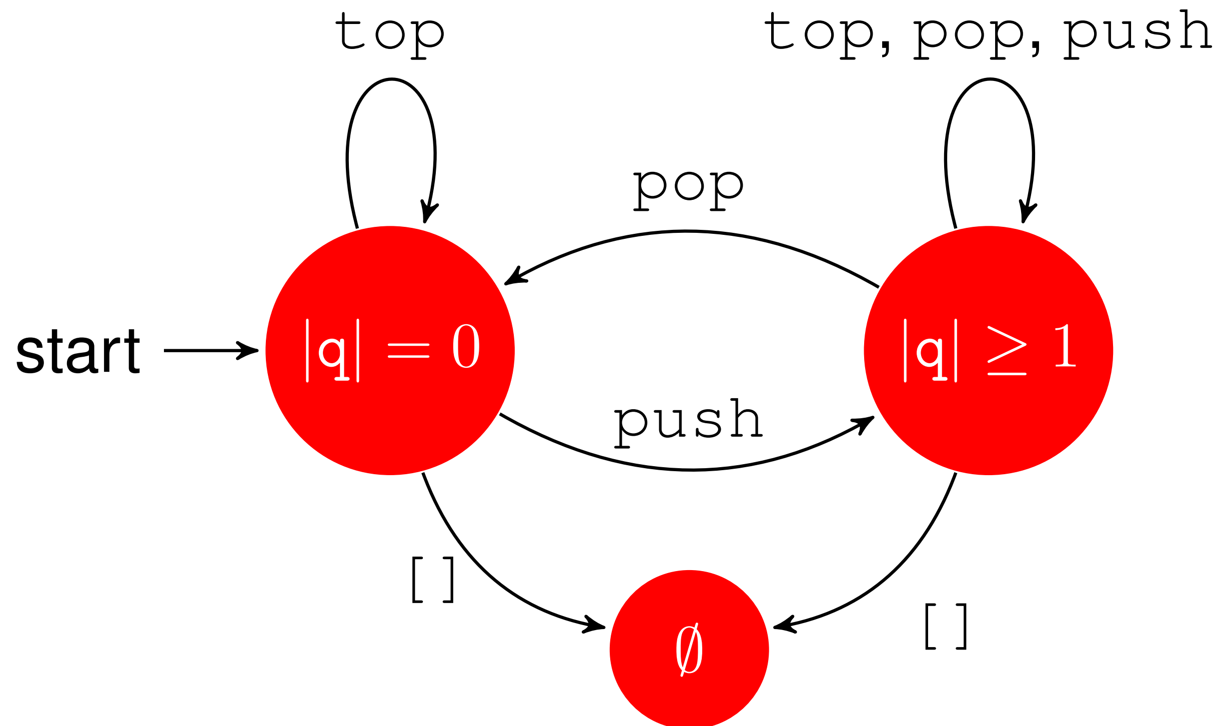
We can now write a full generator, still parameterized:

```
let arb_cmds n =
  make ~print:commands_to_string (commands n)
```

Generator as a state machine

We can think of our generator as a state machine

(the state is the element count)



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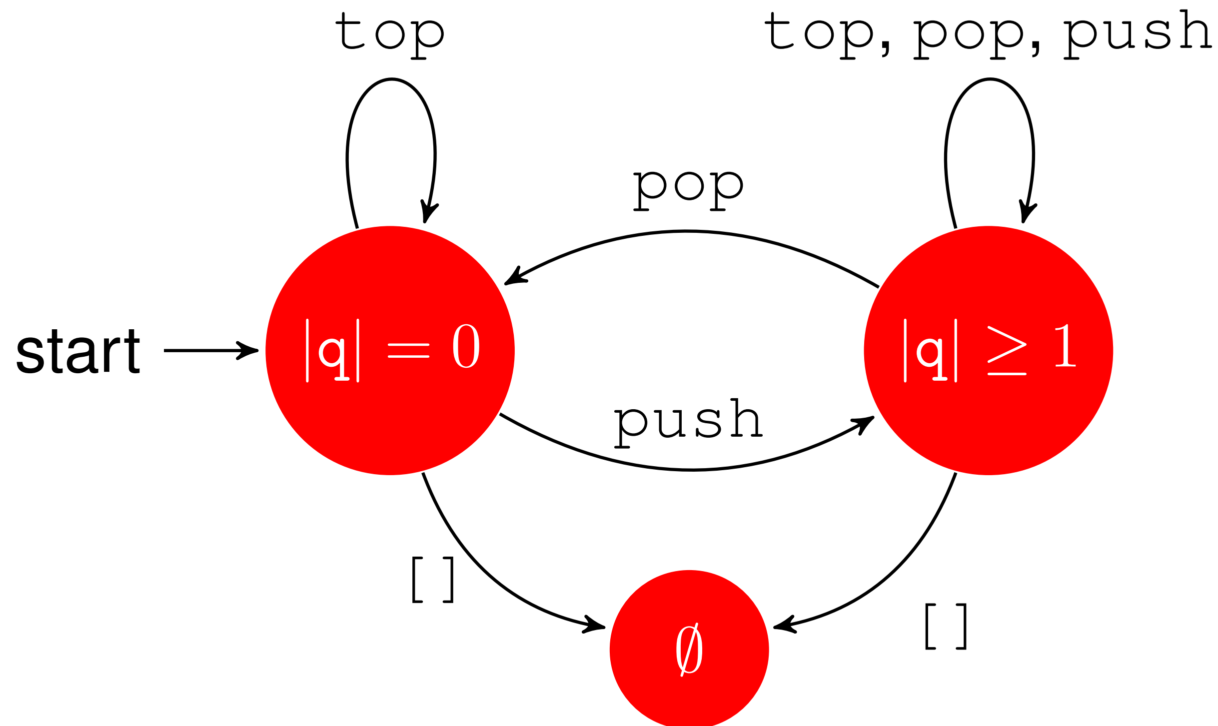

With this kind of specification, `pop`'s precondition "queue is non-empty" can serve two purposes: (1) express an implication property and (2) drive the generator

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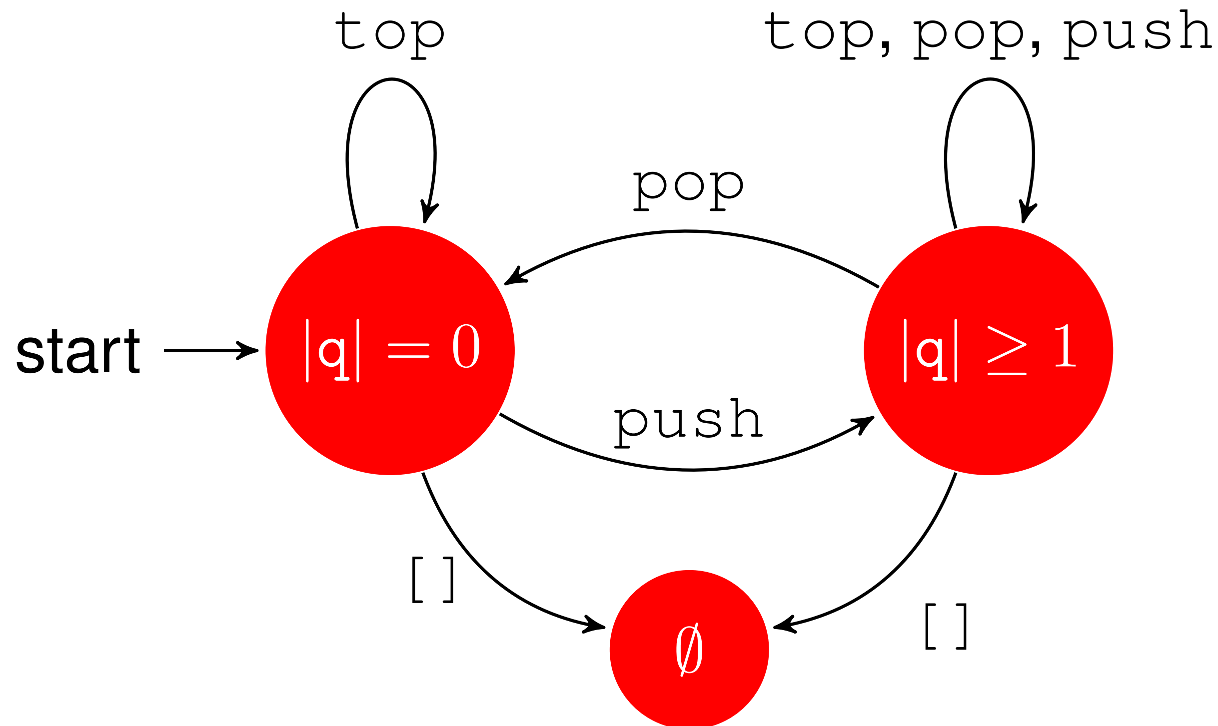

The commercial QuickCheck (for Erlang) comes with a dedicated domain specific language (or library) for writing such "state machine models"

Equivalences beginning in an empty queue

...we can now write tests with this function:

```
let eqempty cmds_pair =
  let cmds,_ = cmds_pair 0 in (*hack to get cmds for 'delta'*)
  Test.make
    (pair (arb_cmds (delta cmds)) int)
    (fun (suffix,m) ->
      let cmds,cmds' = cmds_pair m in
      let observe cmds =
        let q = MyQueue.empty () in
        interp q (cmds @ suffix) in
      observe cmds = observe cmds')
```

which tests that two instruction sequences are observably the same with an arbitrary suffix appended.

We can now write example tests as:

```
eqempty (fun m -> [Top],[Let None])
eqempty (fun m -> [Push m; Top],[Push m; Let (Some m)])
eqempty (fun m -> [Push m; Pop],[])
```

Equivalences in any queue (1/3)

For the equivalences that hold in any queue we need generators that depend on generators

(e.g., suffix depends on prefix)

For this situation a `bind` operation is handy:

```
(>>=) : 'a Gen.t -> ('a -> 'b Gen.t) -> 'b Gen.t
```

It is written as an inline operator `>>=`

For example, we can write a generator of integer pairs, where the first component is less than the second:

```
let my_pair_gen =
  let open Gen in
  small_int >>= (fun i -> pair (int_range 0 i) (return i))
```

(`bind` also occurs in the Haskell, Erlang,... libraries)

Equivalences in any queue (2/3)

For the equivalences that hold for any queue we can write the tests based on >>=:

```
let eq cmds_pair =
  let cmds,_ = cmds_pair 0 0 in (*again: get cmds for delta*)
  let gen_pair =
    make
      ~print:(Print.pair commands_to_string commands_to_string)
      (let open Gen in
       commands 0 >>= fun pref ->
         commands (delta (pref@cmds)) >>= fun suff ->
           return (pref,suff)) in
  Test.make (triple gen_pair int int)
    (fun ((pref,suff),m,n) ->
      let cmds,cmds' = cmds_pair m n in
      let observe cmds =
        let q = MyQueue.empty () in
        interp q (pref @ cmds @ suff) in
      observe cmds = observe cmds' )
```

which tests/observes under arbitrary prefix and suffix.

Equivalences in any queue (3/3)

With `eq` we can now write the last two tests as

```
eq (fun m n -> [Push m; Push n; Top],[Push m; Top; Push n;])
eq (fun m n -> [Push m; Push n; Pop],[Push m; Pop; Push n;])
```

Collectively the end result is not too far notationally from what we were after:

```
QCheck_runner.run_tests ~verbose:true [
  eqempty (fun m -> [Top],[Let None]);
  eqempty (fun m -> [Push m; Top],[Push m; Let (Some m)]);
  eqempty (fun m -> [Push m; Pop],[])
  eq (fun m n -> [Push m; Push n; Top],[Push m; Top; Push n;]);
  eq (fun m n -> [Push m; Push n; Pop],[Push m; Pop; Push n;]);
 ]
```

(it is straightforward to add an optional title argument to `eq` and `eqempty`)

Catching errors

If we introduce an error in the algebraic specification, e.g.

```
eq (fun m n -> [Push m; Push n; Pop],[Push m; Top; Push n;]);
```

QCheck will catch it:

```
generated error fail pass / total     time test name
...
[✗]     1     0     1     0 / 1000     0.0s pushpop

--- Failure --------------------------------------------

Test pushpop failed (123 shrink steps):

(([], [Pop; Top; (Push -2942454129666812723)]), 0, 0)
```

While the last two numbers have been shrunk,
a dedicated command list shrinker could be useful

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Summary

We've seen an approach to test stateful code based on algebraic specifications:

- which are tested for operational equivalence
- requires a generator of command contexts

Again it builds on symbolic representations
– and thereby permits, e.g., shrinking

Next time, we will see how one can also take a model-based approach to test such stateful code