

02913 Advanced Analysis Techniques

QuickCheck, Day 6

Jan Midtgaard

DTU Compute

Outline

- Patrick & Mathias' 2016 Project Presentation
- Exercises from Friday
- Project/exam formalities
- Case study:
 QuickChecking Static Analysis Properties

2016 Project Presentation

Patrick Kasting / Mathias Justesen:
Testing OCaml's Compiler Backends
by Generating Random Lambda Terms

Exercises

Friday's exercises

Project/exam formalities

Project/exam formalities

- **Tuesday, Jan 10**, is the last day of lectures
 - Afterwards you start project work
 - You can do a group project (1-3 members)
 - I'd like to talk briefly to each of you about your project topic
- **Thursday morning, Jan 19**, you hand in a project report (and code)
- **Friday, Jan 20**, you present your projects to the class and receive + answer questions
 - You receive a combined grade for the project report + presentation + answers
 - Measure of success: ability to apply QuickCheck (to a project of your choice)

QuickChecking Static Analysis Properties

This work

Based on a research paper:

QuickChecking Static Analysis Properties

Jan Midtgaard and Anders Møller

8th IEEE International Conference on Software
Testing, Verification and Validation (ICST'15)

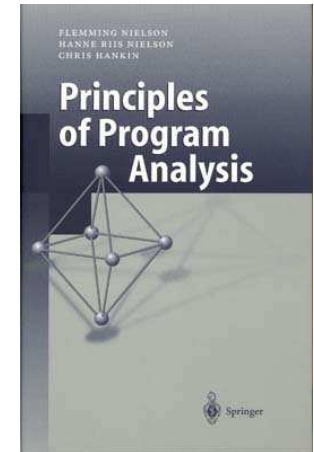
<http://janmidtgaard.dk/papers/Midtgaard-Moeller%3aICST15.pdf>

Static Analysis?

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A *static analysis* predicts properties about a program **without running it**.

Applications:

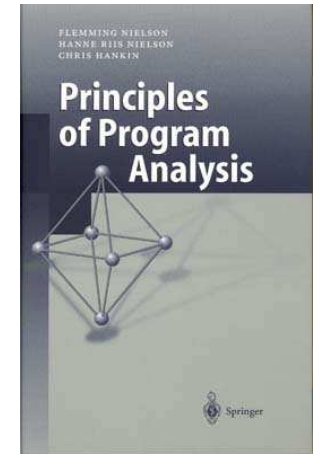


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Applications: Lots

detect run-time errors (null-pointer dereference, array out of bounds, . . .), drive optimization, IDE type feedback, drive refactoring, verify software contracts, . . .



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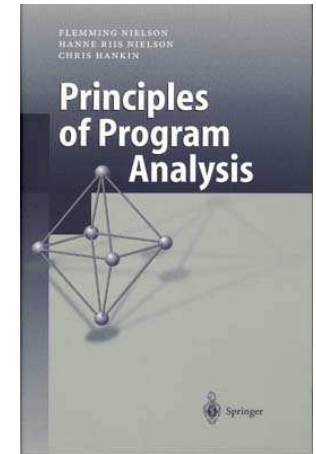
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- so we settle for (sound) approximations that account for all possible executions



Static Analysis? Take 02242!

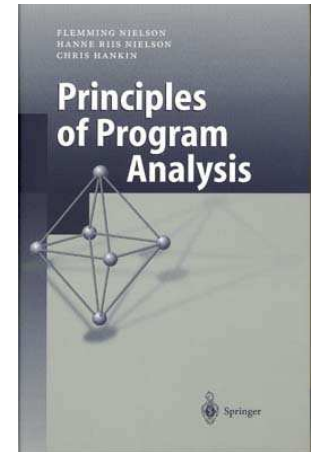
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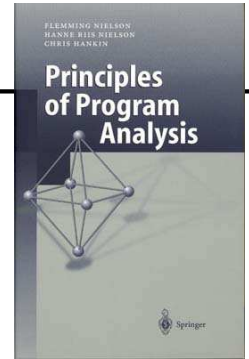


Who tests the tester?

Or rather:
How do we test the tester?

Or rather:
What do we test it for?

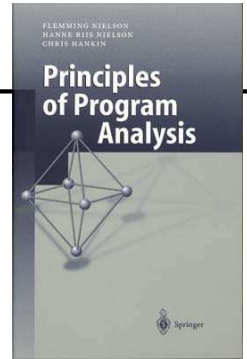
Fact: **Many** static analyses build on...



Theorem (Tarski 1955). Given

- a complete lattice $\langle L; \sqsubseteq \rangle$ and
 - a monotone (order-preserving) function $f : L \rightarrow L$,
- then f 's fixed points ($f(x) = x$) form a complete lattice.
In particular f has a least and a greatest fixed point.

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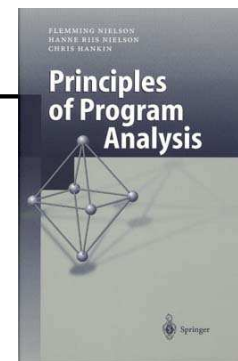


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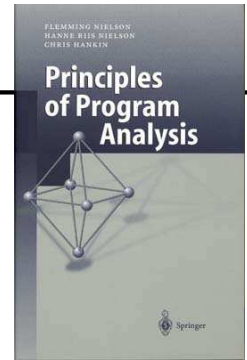
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The properties embody the programmer's intention.

How do we ensure them?

A range of approaches...

for ensuring such *static analysis properties*:

- basic testing (unit test lattices, blackbox test analysis)
- informal monotonicity arguments
- pen-and-paper monotonicity/soundness proofs
- ...
- machine-formalized proofs (Cachera-al:TCS05)
- automated fixed point checking (Blazy-al:SAS13)
- static monotonicity analysis (Murawski-Yi:VMCAI02)

'QuickChecking Static Analysis Properties'

Check essential properties (lattice, monotonicity, . . .)

- with randomized property-based testing
aka QuickCheck (Claessen-Hughes:ICFP'00) and
- an embedded domain specific language (EDSL)

for increased confidence in your static analysis code.

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Pro: lightweight,
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Con: **confidence,**
not proof

A simple example (1/3)

A two-element lattice expressed as an OCaml module:

```
module L = struct
  let name = "example lattice"
  type elem = Top | Bot
  let leq a b = match a,b with
    | Bot, _ -> true
    | _, Top -> true
    | Top, Bot -> false
  let join e e' = if e = Bot then e' else Top
  let meet e e' = if e = Bot then Bot else e'
  (* ... *)

  let to_string e = if e = Bot then "Bot" else "Top"
end
```

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  (* ... *)
  let arb_elem = Arbitrary.among [Bot; Top]
  let to_string e = if e = Bot then "Bot" else "Top"
end
```

We extend the lattice with a **generator** of arbitrary lattice elements.

A simple example (2/3)

Using our reusable functor of lattice property tests

```
# let module LTests = GenericTests(L) in  
  run_tests LTests.suite;;
```

A simple example (2/3)

Using our reusable functor of lattice property tests

```
# let module LTests = GenericTests(L) in
  run_tests LTests.suite;;
  check 19 properties...
testing property leq reflexive in example lattice...
  [✓] passed 1000 tests (0 preconditions failed)
testing property leq transitive in example lattice...
  [✓] passed 1000 tests (0 preconditions failed)
testing property leq anti symmetric in example lattice...
  [✓] passed 1000 tests (0 preconditions failed)
testing property bot is lower bound in example lattice...
  [✓] passed 1000 tests (0 preconditions failed)
testing property join commutative in example lattice...
  [✓] passed 1000 tests (0 preconditions failed)

... (28 additional lines cut)...

tests run in 0.02s
[✓] Success! (passed 19 tests)
```

we can quickcheck \mathbb{L} for a range of lattice properties!

A simple example (3/3)

– and there's more!

Suppose we have a lattice operation:

```
(* flip : L -> L *)  
let flip e = if e = L.Bot then L.Top else L.Bot
```

Note: This guy is **not** monotone.

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Let's test it using our type-safe EDSL
(with mathmode-inspired arrow syntax $\text{flip} : L \xrightarrow{\square} L$)

```
# let flip_desc = ("flip", flip) in  
run (testsig (module L) -<-> (module L) =: flip_desc);;
```

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```
# let flip_desc = ("flip", flip) in  
run (testsig (module L) -<-> (module L) =: flip_desc);;  
testing property 'flip monotone in 1. argument' ...  
[X] 270 failures over 1000 (print at most 1):  
(Bot, Top)
```

The rest of this talk

- Introduction
- QuickCheck, briefly
- A more complex example: Lua type analysis
 - Lua (briefly)
 - A static analysis of Lua
 - Testing lattices and operations
 - Evaluation
- Conclusion

QuickCheck, briefly (1/2)

Throughout we use OCaml with the 'qcheck' library v.0.3

E.g., monotonicity $\forall a, b. a \sqsubseteq b \Rightarrow \text{flip } a \sqsubseteq \text{flip } b$

translates into a function modeling the implication:

```
fun (a,b) -> Prop.assume (L.leq a b); L.leq (flip a) (flip b)
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Combinators for composing generators:

```
let arb_pair = Arbitrary.pair L.arb_elem L.arb_elem
```

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A monotonicity test written out:

```
# let mon_test = mk_test arb_pair  
    (fun (a,b) -> Prop.assume (L.leq a b); L.leq (flip a) (flip b));;  
val mon_test : QCheck.test = <abstr>
```

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    (fun (a,b) -> Prop.assume (L.leq a b); L.leq (flip a) (flip b));;  
    val mon_test : QCheck.test = <abstr>  
# run mon_test;;  
testing property <anon prop>...  
[X] 27 failures over 100
```

QuickCheck, briefly (2/2)

To improve usability 'qcheck' supports more arguments:

```
mk_test ~n:1000 ~pp:pp_pair ~name:"flip monotone"  
  arb_pair (fun (a,b) -> Prop.assume (L.leq a b); L.leq (flip a) (flip b))
```

which will

- name the property,
- run 1000 tests instead, and
- use the pretty printer `pp_pair` defined as

```
let pp_pair = PP.pair L.to_string L.to_string
```

to print counter examples

A more complex example:
Lua and a static type analysis for it

Lua?



Lua, briefly

Lightweight language (few, well-chosen features):

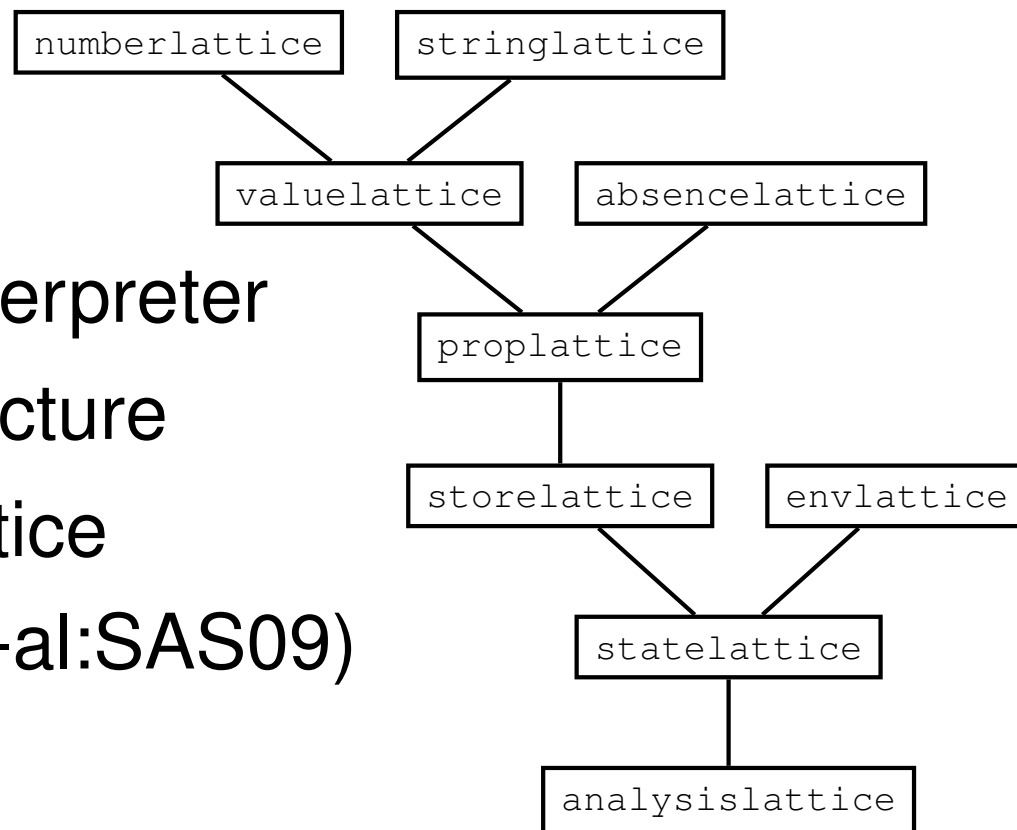
- Dynamically typed
- First-class functions
- Builtin tables (associative arrays)
- ...

Example:

```
1  function mktable(f)
2      return { x = f("x"), y = f("y") }
3  end
4
5  mktable(function (z) return z.." component" end)
```

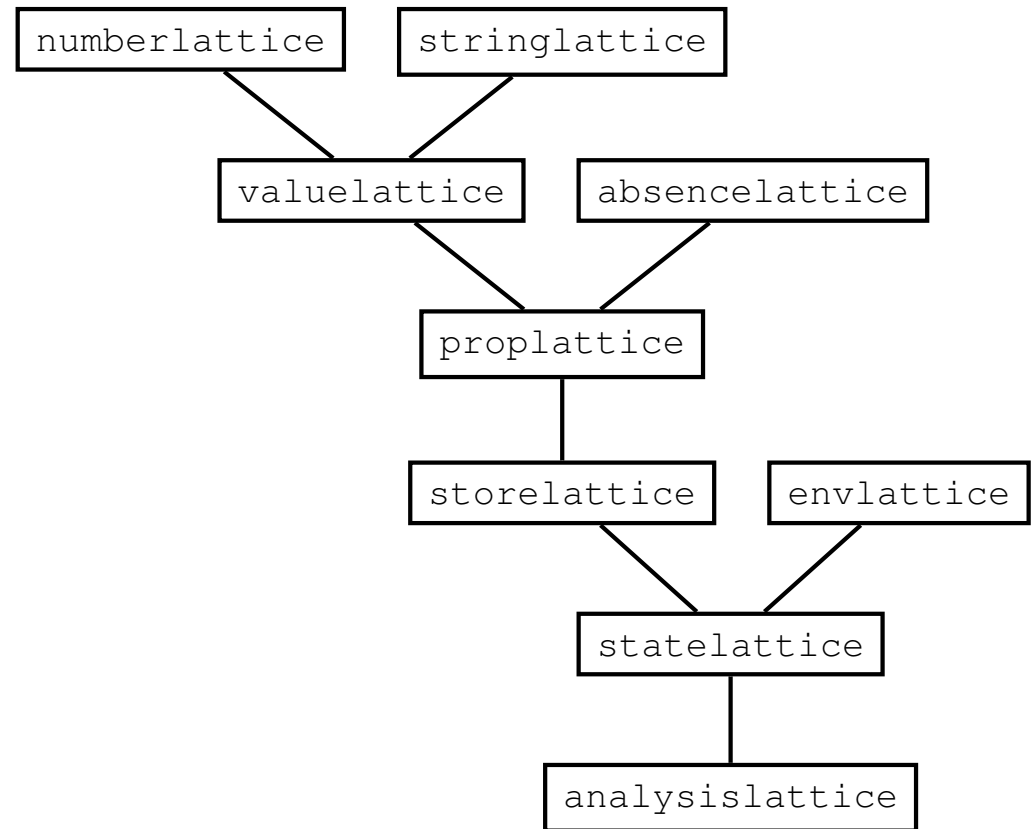
Lua type analysis by example

The static analysis is an approximate (**abstract**) interpreter over a suitable lattice structure (akin to the JavaScript lattice of Jensen-al:SAS09)



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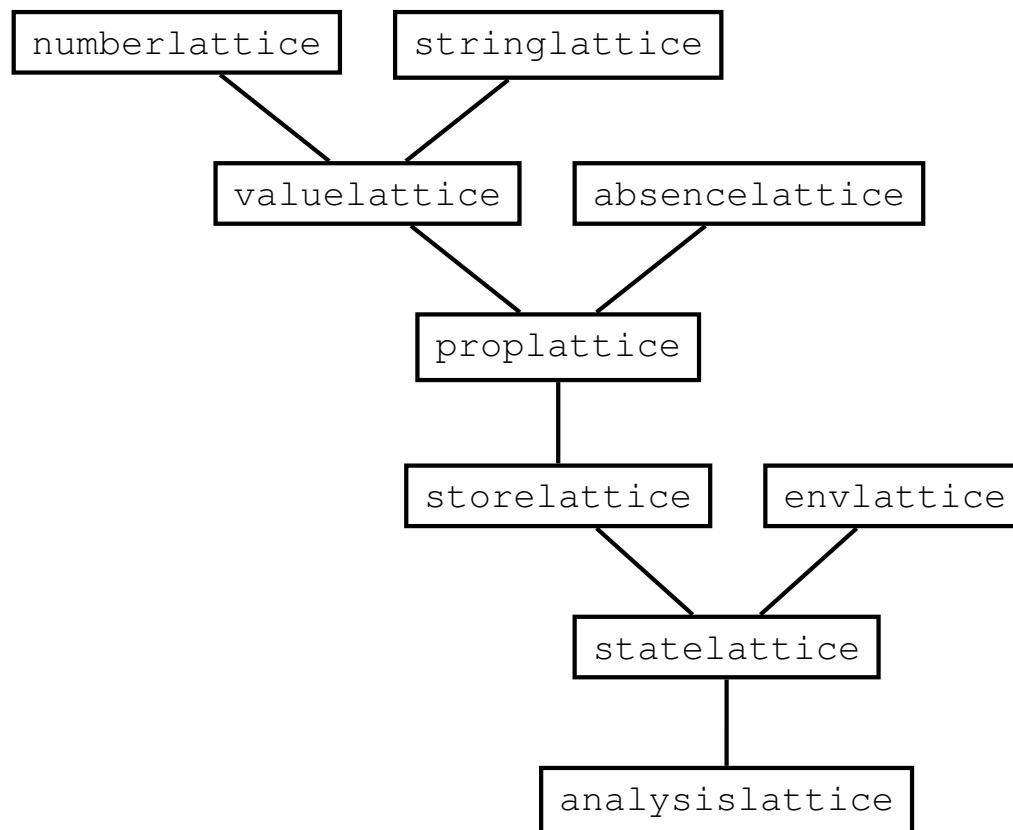
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After this call

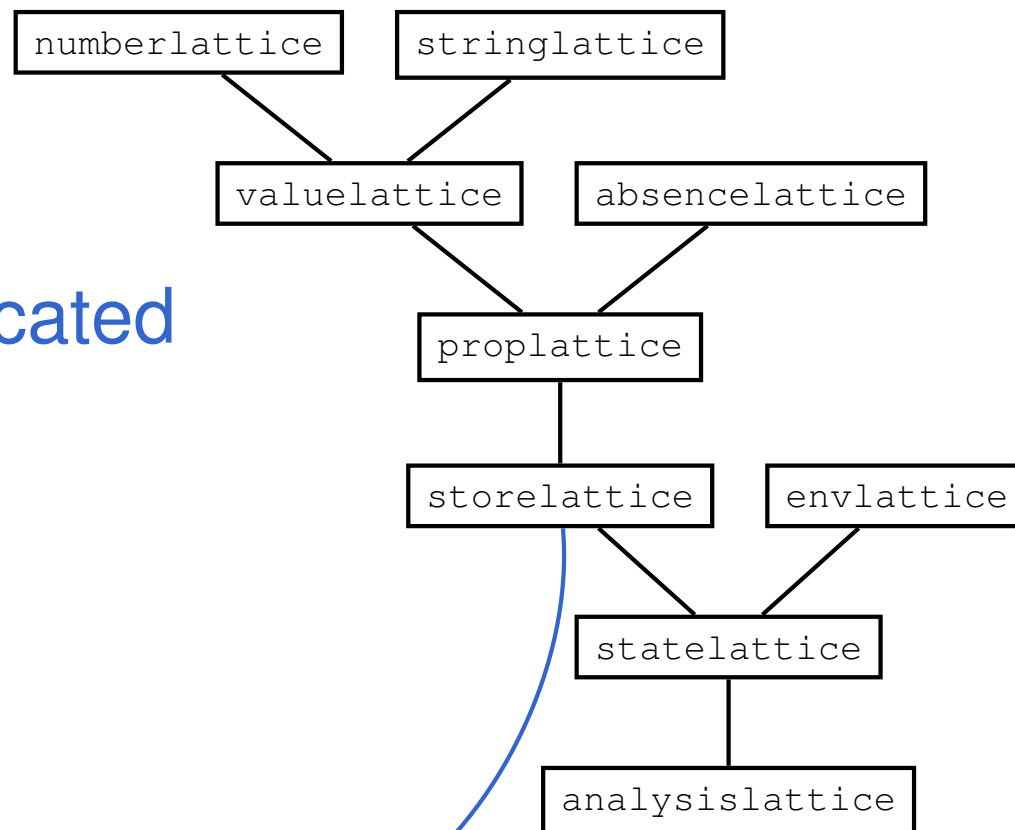


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Lua type analysis by example

After this call

a record originating here
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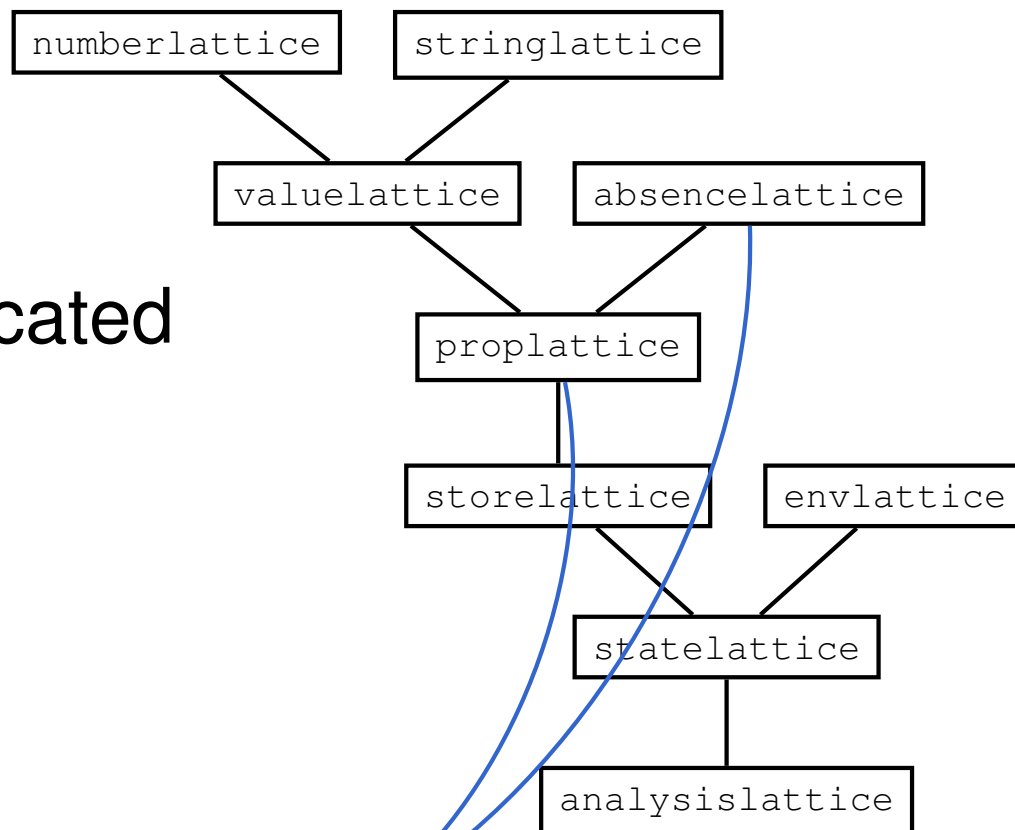
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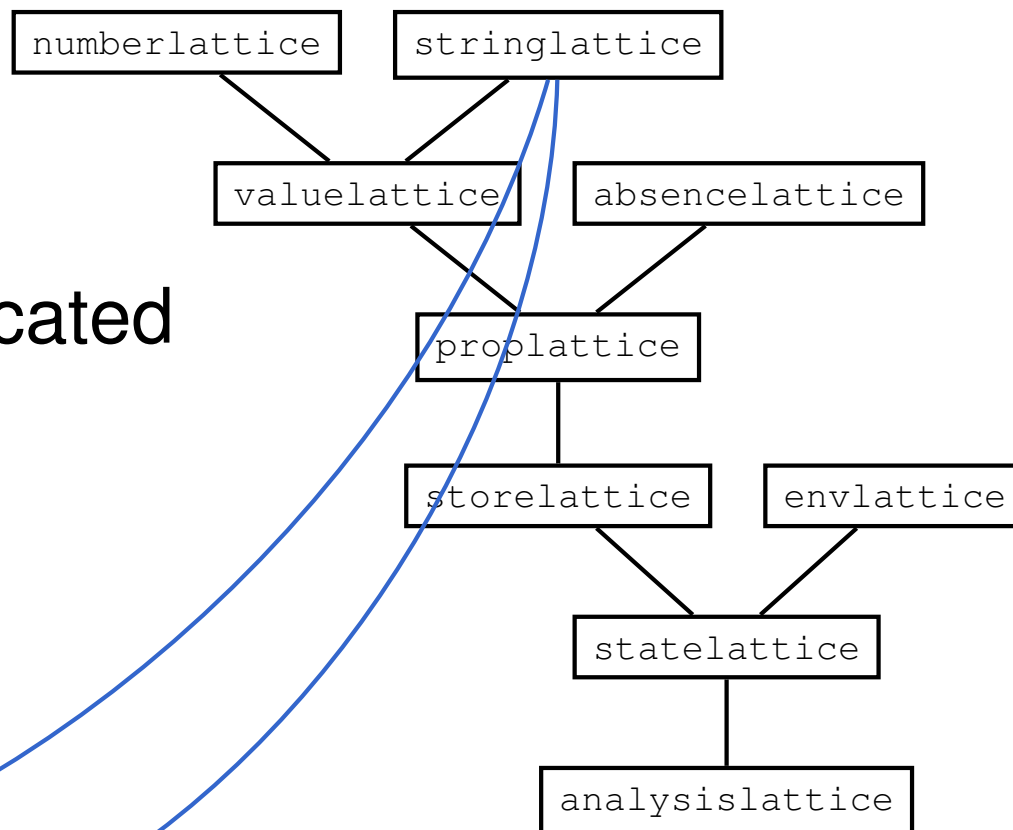
After this call

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has been allocated

which definitely has

entries x and y



```
1 function mktable(f)
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3 end
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Lua type analysis by example

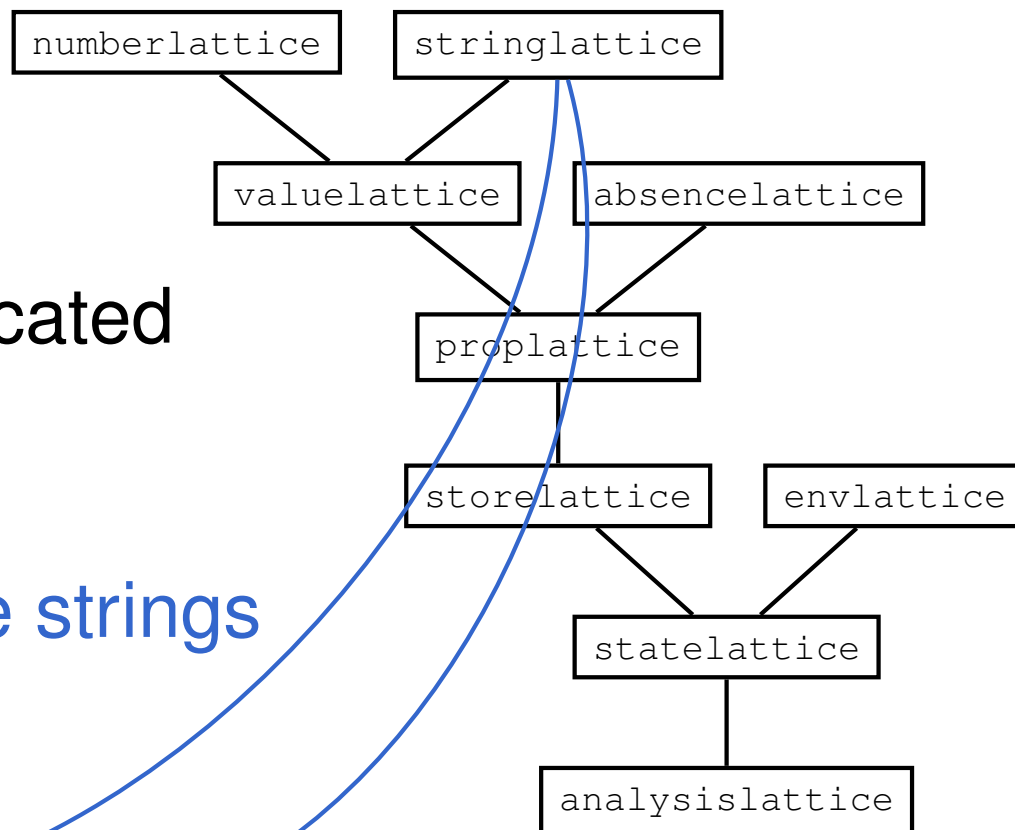
After this call

a record originating here

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which definitely has

entries x and y which are strings



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Lattices for Lua type analysis

The lattice modules have a consistent signature:

```
module type LATTICE_TOPLESS =  
sig  
  type elem  
  val leq      : elem -> elem -> bool  
  val bot      : elem  
  (* val top    : elem *)  
  val join     : elem -> elem -> elem  
  val meet     : elem -> elem -> elem  
  val to_string : elem -> string  
end
```

but not all have an explicit top element

Testing the lattices

Lattice properties

Lattices are partial orders:

$$\forall a \in L. a \sqsubseteq a \quad (\text{reflexivity})$$

$$\forall a, b, c \in L. a \sqsubseteq b \wedge b \sqsubseteq c \Rightarrow a \sqsubseteq c \quad (\text{transitivity})$$

$$\forall a, b \in L. a \sqsubseteq b \wedge b \sqsubseteq a \Rightarrow a = b \quad (\text{anti-symmetry})$$

with additional algebraic properties:

$$\forall a \in L. \perp \sqsubseteq a \wedge a \sqsubseteq \top \quad (\perp \text{ is lower bound, } \top \text{ is upper bound})$$

$$\forall a, b \in L. a \sqcup b = b \sqcup a \wedge a \sqcap b = b \sqcap a \quad (\sqcup, \sqcap \text{ commutative})$$

$$\forall a, b, c \in L. (a \sqcup b) \sqcup c = a \sqcup (b \sqcup c) \wedge (a \sqcap b) \sqcap c = a \sqcap (b \sqcap c) \quad (\sqcup, \sqcap \text{ associative})$$

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So we need `arb_elem` and **equality operation** in signature:

```
val arb_elem : elem Arbitrary.t
```

```
val eq       : elem -> elem -> bool
```

Generating arbitrary lattice elements

Simple generators: (writing `Arb` for `Arbitrary`)

absencelattice: (two-elem lattice)

`Arb.among [Bot; Top]`

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stringlattice: (three-level const-prop lattice)

`Arb.(choose [return bot; lift const string; return top])`

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labelsets: (set-based lattice) Using `qcheck`'s `fix` combinator

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Arb.(fix ~base:(return LabelSet.empty) (lift2 LabelSet.add arb_label))
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Composite generators:

pair lattices: `Arb.pair A.arb_elem B.arb_elem` for lattices `A` and `B`

map lattices: By generating an arbitrary association list:

```
let arb_entries = Arb.(list (pair arb_label pl_arb_elem))
```

and passing the result to a generic helper function:

```
let build_map mt add ls =  
  List.fold_right (fun (k,v) acctbl -> add k v acctbl) ls mt
```

Improving generators

arb_elem isn't always sufficient:

```
let leq_trans = (* forall a,b,c. a <= b /\ b <= c => a <= c *)
  mk_test ~n:1000 ~pp:pp_triple ~name:("leq transitive in " ^ L.name)
    arb_triple (fun (a,b,c) -> Prop.assume (L.leq a b);
                Prop.assume (L.leq b c);
                L.leq a c)
```

as it doesn't lead to very much checking:

```
testing property leq transitive in value lattice...
[✓] passed 1000 tests (1000 preconditions failed)
```

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               Prop.assume (L.leq b c);
               L.leq a c)
```

as it doesn't lead to very much checking:

```
testing property leq transitive in value lattice...
[✓] passed 1000 tests (1000 preconditions failed)
```

Solution: extend signature further:

```
val arb_elem_le : elem -> elem Arbitrary.t
```

to generate elements less than a given argument

Testing the lattice operations

Testing the lattice operations (1/3)

We suggest testing lattice operations for three (five) properties:

- Monotonicity $\forall a, b. a \sqsubseteq b \implies f(a) \sqsubseteq f(b)$

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- **Invariance** $\forall a, b. a = b \implies f(a) = f(b)$

Can help catch “equality related” errors

(cf. Holdermans:PPDP13)

- **Extensiveness** $\forall a. a \sqsubseteq f(a)$ added in 9 LOC

- **Distributivity** $\forall a, b. f(a \sqcup b) = f(a) \sqcup f(b)$ in 10 LOC

Testing the lattice operations (2/3)

Writing things out gets loooong, e.g., for `Str.concat`:

```
(* forall s. bot = s ^ bot *)
let concat_strict_snd =
  mk_test ~n:1000 ~pp:Str.to_string ~name:("Str.concat strict in 2. arg")
    Str.arb_elem (fun s -> Str.(eq bot (concat s bot)))

(* forall s,s',s''. s' <= s'' => (s ^ s') <= (s ^ s'') *)
let concat_monotone_snd =
  mk_test ~n:1000 ~pp:(PP.pair Str.to_string pp_pair) ~name:("Str.concat monotone in 2. arg")
    (Arbitrary.pair Str.arb_elem ord_pair)
    (fun (s, (s', s'')) -> Prop.assume (Str.leq s' s''); Str.(leq (concat s s') (concat s s'')))

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Clearly this will **not scale** to bigger projects

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Props \sim functions, so let's build them with combinators!

Testing the lattice operations (3/3)

For this purpose we offer a combinator-based EDSL

We provide base combinators + 2 for adding arguments:

```
let str_concat = ("Str.concat", Str.concat) in
[ pw_left (module Str) op_strict      (module Str) (module Str) =:: str_concat;
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From combinators to infix arrow syntax:

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Much more compact and type safe

Type safe?

The EDSL is embedded type-safely in OCaml:

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Hence it will catch type errors in the QuickCheck code:

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# let str_concat = ("Str.concat", Str.concat) in  
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but an expression was expected of type

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Type Str.elem = Stringlattice.elem

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How? By (mis)using **continuation-passing style**

Analysis prototype

Functional, written in OCaml (\sim 4700 LOC)

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So far: no quickchecking of tree walker

Initial Findings

- Copy-paste error in meet of `absencelattice`
- Several non-strict lattice operations

(fix improves precision):

```
let unop op lat = match op with
  | Ast.Uminus ->
    let lat' = coerce_tonum lat in
    if may_be_number lat' (* unary minus of number (or better) is number *)
    then number
    else bot
  | Ast.Length ->
    if may_be_strings lat || may_be_table lat
    then number
    else bot (* length of everything but strings and tables is number *)
  | Ast.Not ->
    bool (* negation of anything is bool *)
```

- Not all operations should be strict, e.g., `PL.add`

Quickchecking is a reason to (re)consider properties

A proplattice war story (1/3)

Consider `proplattice` that represents Lua's tables.

```
type elem = { table      : (VL.elem * Abs.elem) TableMap.t;  
              default_key : VL.elem;   (* collective key approx.  *)  
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Another valuelattice approximates the **values** of all statically **unknown entries**

With our **initial generators** this design passed all tests

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(e.g., for testing lookup)

Uniform generators are unlikely to do so

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This led us to reconsider+revise the domain ordering

A proplattice war story (3/3)

The resulting proplattice design reads:

```
type elem =
  | Bot
  | Table of table
and table = { table      : (VL.elem * Abs.elem) TableMap.t;
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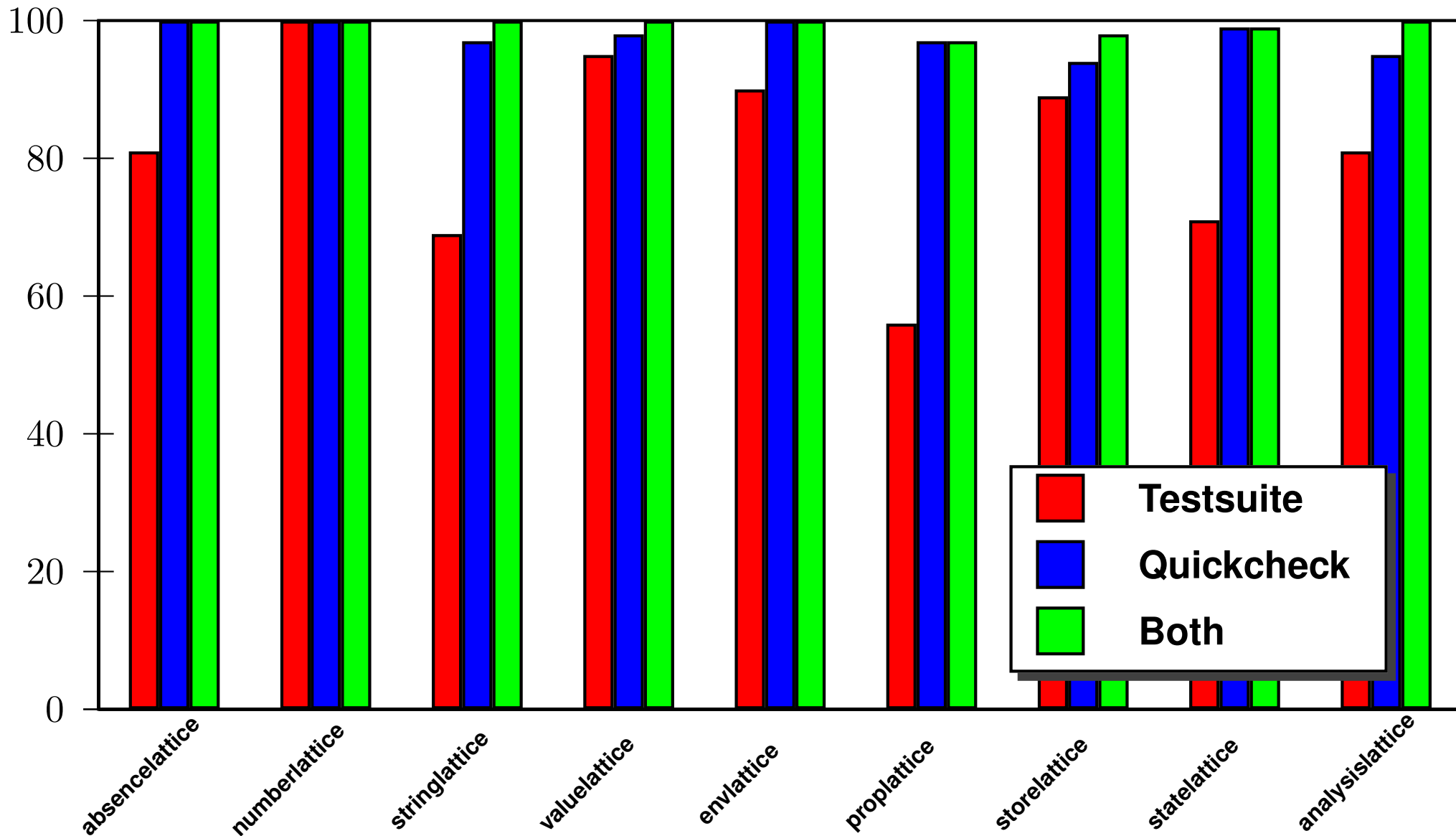
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Fix: 'maybe absent' \perp -entry expresses certain absence!

This revision added an explicit bottom element, so, e.g.,
PL.find is also strict (uniformity)

Experiment: coverage of lattice code (in %)

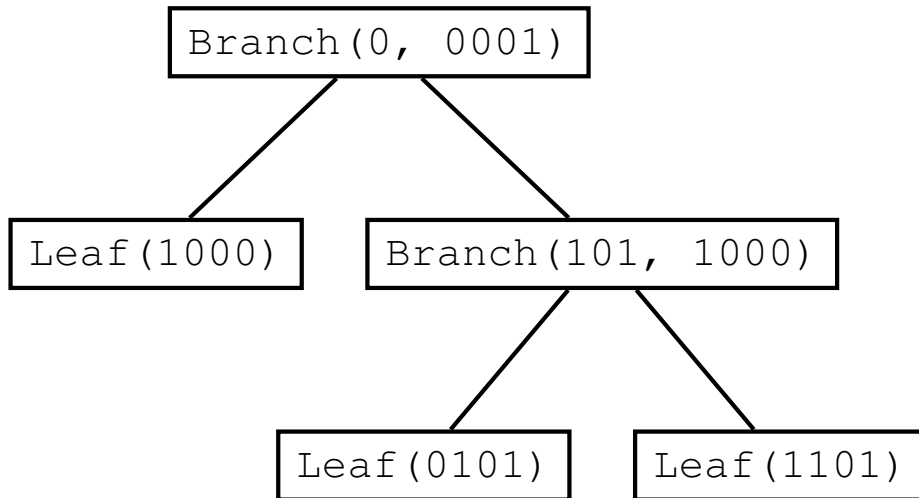


QuickChecking Patricia Trees

Patricia trees (1/5)

A **Patricia tree** is a data structure for representing integer sets (+ maps) **compactly** and **functionally**.

For example, we represent $\{5, 8, 13\}$ as follows:



Branch(`pre`, `brbit`) nodes:

- LSB distinction**
- `pre` is shared prefix
- `brbit` is branching bit

The `ptrees` library is a popular OCaml implementation.

 Since powersets form a lattice, **let's quickcheck** `ptrees`!

Testing Patricia trees (2/5)

We can cook up an interface to LCheck easily:

```
module Ptlat = struct
  let name = "Patricia Tree Intset"
  type elem = Ptset.t
  let leq  = Ptset.subset
  let join = Ptset.union
  let meet = Ptset.inter
  let bot  = Ptset.empty
  let eq   = Ptset.equal
  (* ... *)
  let arb_elem =
    Arbitrary.(fix ~max:8 ~base:(return bot) (lift2 Ptset.add arb_int))
  (* ... *)
end
```

It turns out that the choice of `arb_int` is crucial...

Testing Patricia trees (3/5)

With `arb_int` implemented as a uniform generator:

```
$ ./test.byte
check 19 properties...
testing property leq reflexive in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)
testing property leq transitive in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)
testing property leq anti symmetric in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)
testing property bot is lower bound in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)
testing property join commutative in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)
testing property join associative in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)

[...]

tests run in 0.04s
[✓] Success! (passed 19 tests)
```

The data structure seems to work fine...

Testing Patricia trees (4/5)

“Test corner cases” — 50 years of software eng.

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Hmmm, suppose we either generate one of $\{\text{min_int}, 0, \text{max_int}\}$ or choose uniformly...

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```
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  [✓] passed 1000 tests (0 preconditions failed)
testing property join commutative in Patricia Tree Intset...
  [✓] passed 1000 tests (0 preconditions failed)
testing property join associative in Patricia Tree Intset...
  [X] 52 failures over 1000 (print at most 1):
  ({-4611686018427387904, 0}, {0}, {4611686018427387903})
```

Testing Patricia trees (5/5)

What's wrong?

A helper function recursively merges two Patricia trees by testing for the rightmost branching bit.

An **integer comparison** is insufficient for this test because the **branching bit** and **sign bit** can coincide.

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The bug has lurked in `ptrees` for years which inherited it from the Okasaki-Gill:ML98 paper's code. Jean-Christophe Filliâtre quickly made an elegant fix.

Fast Mergeable Integer Maps*

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Abstract

Finite maps are ubiquitous in many applications, but perhaps nowhere more so than in compilers and other language processors. In these applications, three operations on finite maps dominate all others: *looking up* the value associated with a key, *inserting* a new binding, and *merging* two finite maps. Most implementations of finite maps in functional languages are based on balanced binary search trees, which perform well on the first two, but poorly on the third. We describe an implementation of finite maps with integer keys that performs well in practice on all three operations. This data structure is not new indeed, it is thirty years old this year but it deserves to be more widely known.

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The bug has lurked in `ptrees` for years which inherited it from the Okasaki-Gill:ML98 paper's code. Jean-Christophe Filliâtre quickly made an elegant fix.

Significance?

`ptrees` is used by Javalib library,
which is used by the Sawja library,
which is used by Facebook's Infer tool

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Summary and Conclusion

Lessons learned

1. QuickChecking is a good opportunity to revisit your **specification**
2. Your confidence should be **proportional** to the quality of your generators
3. **Uniform** distributions are useful
 - for programming **non-uniform** distributions
(bugs don't hide uniformly)
4. Machine-generated examples can be HUGE
 - shrinking support is important (but **rare**) to get humanly understandable counterexamples
5. **Statistics** (coverage stats and/or QuickCheck stats) are important to understand what's going on

Summary and conclusion

Randomized property-based testing adds to the static analysis toolbox.

Using our approach we can check

essential static analysis properties

- which are beyond current practice,
- in a lightweight, type-safe manner
- for increased confidence in an implementation
- or simply to test a research idea in the lab.

Summary and conclusion

Randomized property-based testing adds to the static analysis toolbox.

Using our approach we can check

essential static analysis properties

- which are beyond current practice,
- in a lightweight, type-safe manner
- for increased confidence in an implementation
- or simply to test a research idea in the lab.

You are welcome to play with the EDSL:

<https://github.com/jmid/lcheck>

Lattice checking in retrospect

We did API-based quickchecking (module level)

– but with whitebox generators

Blackbox (API-based) generators would be interesting

Going for a **symbolic, algebraic approach instead**, would let us easily manipulate lattice values:

- shrink them
- generate only known “abstract addresses”.

The suggested static analysis project of Apron/Parma Polyhedra Library should be **API-based**

Summary

Today we have covered

- An example project from last year
- The project/exam formalities
- A case study regarding static analysis