Abstract Interpretation: Exercises for day 1

February 2, 2015

1. - Give an example of a transition system that converges (i.e., reaches a stuck or final state in a finite number of steps)
   - Give an example of a transition system that doesn't converge and whose reachable states collecting semantics converges (i.e., reaches a fixed point in a finite number of steps)
   - Give an example of a transition system that doesn't converge and whose reachable states collecting semantics doesn't converge

   Sketch the trace of all three transition system executions and of all three reachable states collecting semantics

2. - Install and setup OCaml (see, e.g., `http://ocaml.org/docs/install.html`)
   - Implement pretty printing functions in OCaml for the reachable states collecting semantics (you'll also need to extend the transition system signature accordingly)
   - Implement in OCaml a function `lfp` that computes the least fixed point of the transition function by Kleene iteration
   - Implement all three examples from exercise 1 in OCaml,
   - Instantiate the reachable states collecting semantics with them, and
   - use `lfp` to confirm your reachable states collecting semantics computations from exercise 1.

3. Prove that

   $$\langle \wp(A \times B); \subseteq \rangle \underset{\alpha}{\overset{\gamma}{\leftleftarrows\!\!\rightrightarrows}} \langle A \to \wp(B); \dot{\subseteq} \rangle$$

   where

   $$\alpha(R) = \lambda a.\{b \mid (a,b) \in R\}$$
   $$\gamma(F) = \{(a,b) \mid b \in F(a)\}$$

   is a Galois connection (using one of the two equivalent definitions)

Bonus challenge

Prove that the two definitions of a Galois connection are equivalent.